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# Twisted supersymmetry in a deformed Wess-Zumino model in $(2 + 1)$ dimensions

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**ABSTRACT:** Non-anticommutative deformations have been studied in the context of supersymmetry (SUSY) in three and four space-time dimensions, and the general picture is that highly nontrivial to deform supersymmetry in a way that still preserves some of its important properties, both at the formal algebraic level (e.g., preserving the associativity of the deformed theory) as well as at the physical level (e.g., maintaining renormalizability). The Hopf algebra formalism allows the definition of algebraically consistent deformations of SUSY, but this algebraic consistency does not guarantee that physical models build upon these structures will be consistent from the physical point of view. We will investigate a deformation induced by a Drinfel'd twist of the  $\mathcal{N} = 1$  SUSY algebra in three space-time dimensions. The use of the Hopf algebra formalism allows the construction of deformed  $\mathcal{N} = 1$  SUSY algebras that should still preserve a deformed version of supersymmetry. We will construct the simplest deformed version of the Wess-Zumino model in this context, but we will show that despite the consistent algebraic structure, the model in question is not invariant under SUSY transformation and is not renormalizable. We will comment on the relation of these results with previous ones discussed in the literature regarding similar four-dimensional constructions.

**KEYWORDS:** Field Theories in Lower Dimensions, Quantum Groups, Superspaces

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## 1 Introduction

The non-commutativity of space-time coordinates (henceforth referred to simply as “non-commutativity”) was initially proposed as an alternative to solve the problem of ultraviolet (UV) divergences in quantum electrodynamics [1]. The first model that appeared in this context was studied by Snyder in 1947 [2], but the idea of non-commutativity was subdued by the success of renormalization theory to deal with UV divergences. For some time, the notion of non-commutative manifolds was mostly developed in the context of mathematics and mathematical physics by Connes, Woronowics and Drinfel’d [3–5], among others, while some physical applications started to be discussed in the beginning of this century [6].

A main motivation for the contemporary interest in the non-commutativity is linked to the idea that in a quantum theory which incorporates gravity, the nature of space-time may change in short distances, near to Planck scales [7, 8]. In string theory, the non-commutativity appears in a natural way in the low energy limit in the presence of a constant background Neveu-Schwarz two-form  $B_{\mu\nu}$  [9], leading to an effective field theory that lives in a space-time where coordinates have nontrivial commutation relations of the form

$$[x^\mu, x^\nu] = (B^{-1})^{\mu\nu} . \quad (1.1)$$

The simplicity of this particular type of non-commutativity, where the commutators of space-time coordinates is a constant tensor, allows the definition of non-commutative versions of known quantum field models in a way that is very well suited for perturbative calculations. This approach became known as *canonical non-commutativity*, and it was extensively studied in [10, 11]. Supersymmetric models with canonical non-commutativity could be easily defined since the deformation given in (1.1) does not interfere with the Grassmannian coordinates of the superspace [12], and it was in fact shown that SUSY was an important ingredient to tame some of the potentially dangerous UV/IR divergences

present in non-commutative models [13–16]. The canonical non-commutativity was even considered in the context of non-relativistic quantum mechanical models, where several interesting effects were unveiled [17–22].

The canonical non-commutativity, however, involves a preferential direction in space-time, given by the constant background tensor in (1.1), inducing an explicit violation of Lorentz invariance. It is an interesting and nontrivial problem to introduce non-commutativity in space-time while still preserving Lorentz invariance: one possibility to do this is by means of the Hopf algebra formalism. In [23], it was shown that although canonical non-commutative theories violate the Lorentz invariance, they respect another closely related symmetry, namely the twisted Lorentz symmetry. In the context of Hopf algebras, the twisted Lorentz symmetry can be understood as a deformation of the standard Poincaré algebra by means of a Drinfel’d twist [5, 24, 25], such that the Poincaré algebra is not deformed, while the co-algebra is. The reader can find some studies in the literature regarding Hopf algebras and deformations associated to Lie algebras in [26–28], for example.

Given the interest in the study of theories with non-commutativity in space-time, it became a natural question to introduce the same ideas in the superspace, looking for supersymmetric models that live in deformed superspaces. Besides the already quoted possibility of the canonical case, in which the supersymmetry structure of the models is left untouched by the deformation, one may entertain the possibility of building non-anticommutative (NAC) models, where the algebra of the Grassmannian coordinates is deformed. General superspace deformations were considered in [29, 30], where it was shown for example that preserving the associativity of the product of superfields can only be achieved in very specific cases.

The possibility of NAC deformations was also shown to appear in the context of superstring theory [31], in which case the non-anticommutativity of the Grassmannian coordinates is associated to the presence of a symmetric constant graviphoton field  $C^{ab}$ , viz.

$$\{\theta^a, \theta^b\} = C^{ab}, \quad \{\bar{\theta}^{\dot{a}}, \bar{\theta}^{\dot{b}}\} = 0, \quad (1.2)$$

where latin indices are indices of two-component spinors. This deformation is only possible in Euclidean space-times, where  $\theta$  and  $\bar{\theta}$  are not related by complex conjugation. Besides, the deformation described by eq. (1.2) breaks half of the original supersymmetry, hence this construction became known as  $\mathcal{N} = 1/2$  SUSY. More explicitly, one may observe that the only anticommutation relation between supercharges that are modified by the background tensor  $C^{ab}$  is

$$\{\bar{Q}_a, \bar{Q}_b\} = -4C^{ab}\sigma_{a\dot{a}}^{\mu}\sigma_{b\dot{b}}^{\nu}\frac{\partial^2}{\partial y^{\nu}\partial y^{\mu}}. \quad (1.3)$$

In practice, the anticommutation relations in eq. (1.2) can be introduced in a given supersymmetric action by replacing the usual product of superfields by the Moyal product

$$f(\theta) \star g(\theta) = f(\theta) \exp\left(-\frac{C^{ab}}{2} \frac{\overleftarrow{\partial}}{\partial\theta^a} \frac{\overrightarrow{\partial}}{\partial\theta^b}\right) g(\theta). \quad (1.4)$$

Using the star product, it is possible to define a deformed Wess-Zumino (WZ) Lagrangian

$$S_{WZ}^* = \int d^4\theta \Phi \bar{\Phi} + \int d^2\theta \left( \frac{1}{2} m \Phi \star \Phi + \frac{1}{3} \Phi \star \Phi \star \Phi \right) + \int d^2\bar{\theta} \left( \frac{1}{2} m \bar{\Phi} \star \bar{\Phi} + \frac{1}{3} \bar{\Phi} \star \bar{\Phi} \star \bar{\Phi} \right). \quad (1.5)$$

In this case the supercharge  $Q$  generates a symmetry of the theory, while  $\bar{Q}$  does not, showing again that only half of the supersymmetry is preserved by (1.2). By expanding the Moyal products in (1.5) in a (finite) power series in  $C^{ab}$ , and then integrating in the Grassmann coordinates to obtain the deformed WZ action in terms of component fields, it can be shown that

$$S_{WZ}^* = S_{WZ} - \frac{1}{3} g \det C \int d^4x F^3, \quad (1.6)$$

where  $F$  is the auxiliary field contained in  $\Phi(y, \theta)$  and  $S_{WZ}$  the undeformed WZ action. This result shows that the NAC in this case amounts to the addition of a single term in the Lagrangian, proportional to  $F^3$ .

The study of the quantum properties of the  $\mathcal{N} = 1/2$  WZ model was reported in [32–34], using the spurion field formalism to include the  $F^3$  term present in eq. (1.6) within the standard superfield formalism. Renormalization was shown to be possible but quite nontrivial, since a finite number of additional counterterms have to be included in the theory to absorb UV divergences of the quantum effective action.

In  $(2+1)$  dimensions, the structure of supersymmetric models is simpler in a sense, since the Grassmann coordinates  $\theta$  are real (due to the Lorentz group being related to  $SL(2, \mathbb{R})$  instead of  $SL(2, \mathbb{C})$  as in  $(3+1)$  dimensions), and the notion of chirality is absent. This simplicity, however, seems to leave less room to define deformations of supersymmetry that still preserve at least some of the interesting properties of supersymmetric models. Some options were first discussed in [35], where the strategy was to start with  $\mathcal{N} = 2$  SUSY, which was brought down to  $\mathcal{N} = 1$  by the deformation. The problem of deformation of  $\mathcal{N} = 2$  three-dimensional SUSY have been revisited recently [36–38], mostly motivated by the search of deformed variants of ABJM theories [39].

In this paper, we want to investigate one alternative way to introduce NAC in three-dimensional SUSY that has not yet been developed, which is the use of twisted symmetries. In the case of four-dimensional SUSY, there was some extensive work investigating several possible deformations [40–42]. However, it was already pointed out that algebraic consistency is not enough to ensure the construction of physically meaningful models. In [43–46], two different twist deformations of four-dimensional SUSY were thoroughly examined, and it was found that one may write actions in terms of superfields that, despite being formally covariant, are actually not invariant under SUSY transformations. Even if this problem can be circumvented in order to define a deformed WZ model at the classical level, investigation of the quantum corrections showed that it turn out to be non-renormalizable. The need to preserve the notion of chirality (by means of the introduction of non-linear projection operators) was at the core of the problems reported in [43–46], so one might wonder whether the situation in three-dimensional case, where there is no chirality, could be better.

The formalism of twist deformations would be particularly interesting in three dimensions because it could potentially implement NAC in an  $\mathcal{N} = 1$  supersymmetric model,

since on the Hopf algebraic description, one may deform only the co-algebra and not the algebra itself. An undeformed SUSY algebra would mean, in principle, a NAC model invariant under the same SUSY transformations as the undeformed one, but with a deformed Leibnitz rule when the supercharges act on the product of superfields. However, we will show that the same problems found in four-dimensional models appear here: the component version of the most natural definition of a deformed WZ model fails to be supersymmetric invariant, and when quantum corrections are calculated using the superfield formalism, the model turns out to be non-renormalizable. The end result is that SUSY in  $(2+1)$  dimensions, despite being somehow simpler in structure, seems to impose stringent restrictions on the possible NAC deformations one can consistently define, even when the algebraic machinery of Hopf algebras is used.

This paper is organized as follows. In section 2, we give a brief review of SUSY in the language of Hopf algebras (for a more detailed review, see the discussion of the four-dimensional case in [42, 43], for example). The deformation of the supersymmetry algebra using a twist element is introduced in section 3. In section 3 we define a covariant field theory under the deformed algebra, which would be the deformed version of the WZ model. We discuss the basic properties of this model, showing how despite being written in terms of covariant superfields and operators, the action fails to be invariant under component supersymmetry transformations. Regardless this issue, the model can be quantized and its renormalization studied, by using the spurion technique, and in section 5 we show that the models turns out to be non-renormalizable. Section 6 contains our conclusions. The notations and conventions of [47] are used throughout the text.

## 2 Supersymmetry in the Hopf algebra formalism

The supersymmetric Poincaré superalgebra has an universal enveloping algebra which has the natural structure of a Hopf superalgebra. This construction preserves the main properties of the usual SUSY algebra, such as the anticommutators and the Jacobi identities [48]. Indeed, the SUSY algebra in  $(2+1)$  dimensions, which will be denoted by  $\mathcal{SP}$  is given by,

$$[P_{ab}, P_{cd}] = 0, \quad (2.1a)$$

$$\{Q_a, Q_b\} = 2P_{ab}, \quad (2.1b)$$

$$[Q_a, P_{cd}] = 0. \quad (2.1c)$$

Here,  $P_{ab}$  is the generator of space-time translations, represented as a bispinor. The universal enveloping algebra  $\mathcal{U}(\mathcal{SP})$  is defined as the quotient of the sum of tensor products of the original algebra  $\mathcal{SP}$  by the ideal generated by the (anti)commutations relations (2.1). The Hopf algebra structure of  $\mathcal{U}(\mathcal{SP})$  is encoded by the coproduct, product and antipode given by

$$\Delta(\zeta) \equiv \sum_i (\zeta_1)_i \otimes (\zeta_2)_i = \zeta \otimes 1 + 1 \otimes \zeta, \quad (2.2)$$

$$\mu(\zeta \otimes \eta) = \zeta \cdot \eta, \quad (2.3)$$

$$S(\zeta) = -\zeta, \quad S(1) = 1, \quad \eta \quad (2.4)$$

where in eq. (2.2) we have used the Sweedler notation [49], and  $\zeta, \eta \in \mathcal{U}(\mathcal{SP})$ . The algebraic structure (2.1) is encoded in the adjoint action of one operator  $\zeta$  into another  $\eta$ , i.e.,

$$\begin{aligned} ad_{\zeta} &= (-1)^{\kappa(\eta)\kappa(\zeta_2)} \zeta_1 \cdot \eta \cdot S(\zeta_2) \\ &= \zeta \cdot \eta - (-1)^{\kappa(\eta)\kappa(\zeta)} \eta \cdot \zeta, \end{aligned} \quad (2.5)$$

where  $\kappa$  is the usual parity function.

The superalgebra  $\mathcal{U}(\mathcal{SP})$  act on superfields, which are functions of the superspace with coordinates  $z = (x^{ab}, \theta^a)$  satisfying

$$[x^{mn}, x^{rs}] = [x^{mn}, \theta^a] = 0, \quad (2.6)$$

$$\{\theta^a, \theta^b\} = 0. \quad (2.7)$$

This action can be represented by first order differential operators as follows,

$$Q_a = i \left( \partial_a - i \theta^b \partial_{ba} \right), \quad (2.8a)$$

$$P_{ab} = i \partial_{ab}, \quad (2.8b)$$

$$D_a = \partial_a + i \theta^b \partial_{ba}, \quad (2.8c)$$

where  $D_a$  are the supercovariant derivatives, which are essential in the definition of covariant supersymmetric actions.

The superfield themselves encompass an algebra with product  $m$ , which in the standard (undeformed) case is given by the pointwise product

$$m(\Phi \otimes \Psi) = \Phi(z) \cdot \Psi(z). \quad (2.9)$$

In the Hopf algebra formalism, the Leibniz rule is represented by the covariant action of the Hopf algebra on the algebra of the superfields, i.e.,

$$\zeta \triangleright (m(\Phi \otimes \Psi)) = m(\Delta(\zeta) \triangleright (\Phi \otimes \Psi)), \quad (2.10)$$

which, for the undeformed coproduct (2.2), reduces the usual Leibnitz rule,

$$\zeta(\Phi \cdot \Psi) = \zeta(\Phi) \cdot \Psi + (-1)^{\kappa(\zeta)\kappa(\Psi)} \Phi \cdot \zeta(\Phi). \quad (2.11)$$

Superfields can be decomposed in terms of component fields. For the simplest case of a scalar superfield  $\Phi(x, \theta)$ , we have

$$\Phi(x, \theta) = A(x) + \theta^a \psi_a(x) - \theta^2 F(x), \quad (2.12)$$

where  $A$  and  $\psi$  are scalar and spinorial fields, and  $F$  is an auxiliary field. SUSY transformation are generated by the supercharges  $Q$ ,

$$\delta_{\xi} \Phi(x, \theta) \equiv i \xi^a Q_a \Phi(x, \theta), \quad (2.13)$$

which in terms of component fields amounts to,

$$\delta A(x) = -\xi^a \psi_a(x) , \quad (2.14)$$

$$\delta \psi_a(x) = -\xi^b (\epsilon_{ab} F(x) + i \partial_{ab} A(x)) , \quad (2.15)$$

$$\delta F(x) = -i \xi^a \partial_a^b \psi_b(x) . \quad (2.16)$$

In the undeformed case,  $Q$  is represented by a first order differential operator, which satisfies the standard Leibnitz rule when applied on product of superfields,

$$\delta_\xi (\Phi \cdot \Psi) = \delta_\xi (\Phi) \cdot \Psi + \Phi \cdot \delta_\xi (\Psi) , \quad (2.17)$$

which, in the Hopf algebra formalism, is encoded by the standard coproduct (2.2).

### 3 Twist deformation of the SUSY algebra

In the Hopf algebra formalism, the deformation can be introduced as a Drinfel'd twist [5, 50, 51]. One starts by choosing a twist element, which we postulate is given by

$$\mathcal{F} = f^a \otimes f_a = \exp \left[ \frac{1}{2} C^{ab} \partial_a \otimes \partial_b \right] , \quad (3.1a)$$

$$\mathcal{F}^{-1} = \bar{f}^a \otimes \bar{f}_a = \exp \left[ -\frac{1}{2} C^{ab} \partial_a \otimes \partial_b \right] , \quad (3.1b)$$

where  $C^{ab}$  is a symmetric matrix.<sup>1</sup> The twist element (3.1) can be shown to satisfy the 2-cocycle condition

$$\mathcal{F}(\Delta \otimes id) \mathcal{F} = \mathcal{F}(id \otimes \Delta) \mathcal{F} , \quad (3.2)$$

which guarantees associativity of the construction. This is similar to the twist considered in [43, 46].

Since the Grassmanian derivatives are nilpotent and anticommutative, when expanded in powers of  $C$  we find  $\mathcal{F}$  to be finite,

$$\mathcal{F} = 1 \otimes 1 + \frac{1}{2} C^{ab} \partial_a \otimes \partial_b - \frac{1}{8} C^{ab} C^{mn} \partial_a \partial_m \otimes \partial_b \partial_n , \quad (3.3)$$

such as in the  $\mathcal{N} = 1/2$  SUSY [31], and in the twisted supersymmetry model studied in [43, 46], both in four space-time dimensions, but differently from the three-dimensional deformation considered in [35], in which the expansion of the Moyal product has infinite terms. It is also interesting to stress that, differently from what happens in four dimensions, a similar twist involving the supercovariant derivative  $D_\alpha$  instead of  $Q_\alpha$  would not be finite, since the  $D$ 's do not anticommute among themselves, so we do not consider a “D-deformation” as in [44, 45], which would be much more complicated in our case.

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<sup>1</sup>One clarification is in order here: to use this particular twist, one has to enlarge the superalgebra  $\mathcal{U}(\mathcal{SP})$  by including the Grassmanian derivatives  $\partial_a$  as new generators, in equal footing to  $Q_a$ ,  $D_a$  and  $P_{ab}$ . This can be done without spoiling the algebraic consistency of the construction.

The deformation is implemented on the algebra of superfields by means of the deformed star product given by

$$\begin{aligned}\Phi(z) \star \Psi(z) &= m^{\mathcal{F}}(\Phi \otimes \Psi) = m(\mathcal{F}^{-1} \triangleright (\Phi \otimes \Psi)) \\ &= (-1)^{\kappa(\Phi) \kappa(\bar{f}_a)} (\bar{f}^a \triangleright \Phi) \cdot (\bar{f}_a \triangleright \Psi) \\ &= \Phi \cdot \Psi - \frac{1}{2} (-1)^{\kappa(\Phi)} C^{ab} \partial_a \Phi \cdot \partial_b \Psi - \frac{1}{8} C^{ab} C^{mn} \partial_a \partial_m \Phi \cdot \partial_b \partial_n \Psi.\end{aligned}\quad (3.4)$$

The start product introduce NAC in the superspace, since

$$[x^{mn} \star x^{rs}] = [x^{mn} \star \theta^a] = 0, \quad \{\theta^a \star \theta^b\} = C^{ab}, \quad (3.5)$$

where these  $\star$  (anti)commutators are defined by replacing the usual product of functions by the star product.

In the context of  $\mathcal{N} = 1/2$  SUSY, the deformation of the Poincaré superalgebra is obtained by formally calculating the anticommutators of the generators (2.8), taking into account eq. (3.5). This would lead to

$$\{Q_a, Q_b\}_\star = 2P_{ab} - C^{mn} P_{ma} P_{nb}, \quad (3.6)$$

$$\{D_a, D_b\}_\star = 2P_{ab} + C^{mn} P_{ma} P_{nb}, \quad (3.7)$$

$$\{Q_a, D_b\}_\star = -i C^{mn} P_{ma} P_{nb}, \quad (3.8)$$

where the breaking of SUSY becomes manifest. However, in the Hopf algebra formalism, this is actually not correct since the star product is, at this point, defined only for superfields, and not for operators. To properly define the star (anti)commutators, we extend the discussion presented in [52] for the case of graded superalgebras. We define the star product between two elements of the superalgebra  $\mathcal{U}(\mathcal{SP})$  as

$$\zeta \star \eta = \sum_a (-1)^{\kappa(\bar{f}_a) \kappa(u)} \bar{f}^a(\zeta) \cdot \bar{f}_a(\eta), \quad (3.9)$$

where  $\bar{f}^a(\zeta) \equiv ad_{\bar{f}_a}(\zeta)$ , the adjoint action defined in (2.5). With this definition, one can calculate for example

$$\begin{aligned}\{Q_a \star Q_b\} &= Q_a \star Q_b + Q_b \star Q_a \\ &= 2P_{ab} - C^{mn} P_{ma} P_{nb},\end{aligned}\quad (3.10)$$

reproducing the result in eq. (3.6). The same can be done for the other generators.

In the context of twisted deformations, for any of the operators  $\eta$  that generate  $\mathcal{U}(\mathcal{SP})$ , one may associate a new (deformed) generator  $\tilde{\eta}$ ,

$$\tilde{\eta} = \sum_a (-1)^{\kappa(\bar{f}_a) \kappa(\eta)} \bar{f}^a \cdot \eta \cdot S(\bar{f}_a), \quad (3.11)$$

also belonging to  $\mathcal{U}(\mathcal{SP})$ , in such a way that  $\tilde{\eta}$  satisfies the original, undeformed algebra [52]. This is a clear advantage of this formalism. For example, considering the supersymmetric generator  $Q_a$ , we have

$$\tilde{Q}_a = Q_a + \frac{i}{2} C^{lm} \partial_m P_{al}, \quad (3.12)$$



and one can verify that

$$\left\{ \tilde{Q}_a, \tilde{Q}_b \right\} = 2P_{ab}. \quad (3.13)$$

Clearly,  $\tilde{Q}_a$  is not linear in the generators of the algebra, so indeed it fits only within the Hopf algebraic machinery, and not the usual Lie algebra formalism. If we consider  $\tilde{Q}_a$  as the generator of supersymmetry transformations, we can say we constructed a deformed NAC superspace, while still preserving supersymmetry.

It is interesting to point out that the possibility of defining nonlinear generators which satisfy the undeformed algebra was already briefly pointed out in different contexts [31, 35]. In [35], for example, it was considered an  $\mathcal{N} = 2$  three-dimensional superspace with real Grassmanian coordinates  $\theta_a^{1,2}$ , and the deformed algebra

$$\{\theta_a^1, \theta_b^1\} = 0, \quad \{\theta_a^2, \theta_b^2\} = \Sigma_{ab}, \quad (3.14)$$

where  $\Sigma_{ab}$  is the deformation parameter. In this formalism, the SUSY transformation generated by  $Q_a^1$  is preserved, while the one generated by  $Q_a^2$  is lost, as can be seen by inspecting their anticommutation relations,

$$\{Q_a^1, Q_b^1\} = 2P_{ab}, \quad \{Q_a^1, Q_b^2\} = 0, \quad \{Q_a^2, Q_b^2\} = 2P_{ab} + \Sigma^{cd} P_{ac} P_{bd}. \quad (3.15)$$

One may however define the nonlinear generators

$$\tilde{Q}_a^2 = Q_a^2 + \frac{i}{2} \Sigma^{bc} \partial_b^2 P_{ca}, \quad (3.16)$$

which satisfy the usual (undeformed) supersymmetry algebra,

$$\left\{ \tilde{Q}_a^2, \tilde{Q}_b^2 \right\} = 2P_{ab}, \quad (3.17)$$

but this possibility was not fully developed in [35] inasmuch as those new generators were represented by nonlinear operators. We see that the deformed generators given in eq. (3.16) fit naturally within the Hopf algebra formalism, realizing an instance of twisted supersymmetry.

The coproduct for the deformed generators is defined by

$$\Delta_\star(\tilde{\eta}) = \mathcal{F}(\tilde{\eta} \otimes 1 + 1 \otimes \tilde{\eta}) \mathcal{F}^{-1}, \quad (3.18)$$

which is compatible with the star product in eq. (3.4), meaning that

$$\tilde{\eta} \triangleright (m^{\mathcal{F}}(\Phi \otimes \Psi)) = m^{\mathcal{F}}(\Delta_\star(\tilde{\eta}) \triangleright (\Phi \otimes \Psi)).$$

The action of the deformed generators on a single superfield can be, in a slight abuse of notation, defined as

$$\tilde{\eta} \triangleright \Phi = \eta \triangleright \Phi, \quad (3.19)$$

meaning the action of the operator  $\tilde{\eta}$  mimics that of the undeformed generator  $\eta$  when acting on a single superfield [43, 51]. The action of  $\tilde{\eta}$  on a product of superfields is deformed

according to the coproduct (3.18). For the supersymmetry generators, this means that the SUSY transformation of a single superfield is undeformed,

$$\delta_\xi^* \Phi(x, \theta) \equiv i\xi^a \tilde{Q}_a \triangleright \Phi(x, \theta) = i\xi^a Q_a \Phi(x, \theta). \quad (3.20)$$

However, due to the deformed coproduct given in eq. (3.18), the SUSY transformation of a star product of superfields is modified according to

$$\delta_\xi^* (\Phi \star \Psi) = (\delta_\xi^* \Phi) \star \Psi + \Phi \star (\delta_\xi^* \Psi) + \frac{i}{2} C^{mn} \xi^a \left( \partial_m \Phi \star \partial_{na} \Psi - (-1)^{\kappa(\Phi)} \partial_{ma} \Phi \star \partial_n \Psi \right). \quad (3.21)$$

In essence, in this formalism the effect of the deformation is to modify the Leibnitz rule according to which supercharges act on products of superfields, while the algebra of supercharges itself is not modified. This formalism should allow us to define actions involving superfields that are, in principle, covariant under SUSY transformations. In concrete terms, when projecting a superfield action in terms of component fields to verify the SUSY invariance, the non invariant terms arising from the star product in  $(\delta_\xi^* \Phi) \star \Psi + \Phi \star (\delta_\xi^* \Psi)$  should cancel, or combine to form surface terms, with the additional terms arising from the order  $C$  correction present in eq. (3.21). As we will show in the next section, however, this does not happen for a simple non trivial deformed model that we study.

#### 4 Deformed Wess-Zumino action

We consider the deformed WZ action in  $(2+1)$  dimensions,

$$\begin{aligned} \mathcal{S}^* &= -\frac{1}{4} \int d^5 z \left( \tilde{D}^b \triangleright \Phi \right) \star \left( \tilde{D}_b \triangleright \Phi \right) \\ &\quad + \frac{1}{2} \int d^5 z m \Phi \star \Phi + \frac{\lambda}{6} \int d^5 z \Phi \star \Phi \star \Phi, \end{aligned} \quad (4.1)$$

where  $\int d^5 z \equiv \int d^3 x d^2 \theta$ . This is the standard WZ action, with the usual products replaced by the star products defined in eq. (3.4). Using integration by parts, one may show from eq. (3.4) that

$$\int d^5 z \mathcal{H} \star \mathcal{G} = \int d^5 z \mathcal{H} \mathcal{G}, \quad (4.2)$$

where  $\mathcal{H}, \mathcal{G}$  are arbitrary superfields. From this property, together with eq. (3.19), we can show that the quadratic term in eq. (4.1) remains undeformed,

$$\mathcal{S}_{\text{kin}}^* = \frac{1}{2} \int d^5 z \left[ \Phi D^2 \Phi + m \Phi^2 \right]. \quad (4.3)$$

As for the cubic interaction terms they reduce to the usual WZ interactions together with an additional term,

$$\mathcal{S}_I^* = \frac{\lambda}{6} \int d^5 z \Phi \star \Phi \star \Phi = \frac{\lambda}{6} \int d^5 z \Phi \Phi \Phi + \frac{\lambda}{48} \int d^5 z C^{lm} C^{nk} \partial^2 \Phi \partial_l \partial_n \Phi \partial_k \partial_m \Phi, \quad (4.4)$$

or, in terms of component fields,

$$\mathcal{S}_I^* = \int d^3 x \mathcal{L}_I + \frac{\lambda}{24} \int d^3 x C^2 F^3, \quad (4.5)$$

where

$$\det C = C^2 = \frac{1}{2} C^{ml} C^{nk} \epsilon_{ln} \epsilon_{mk}, \quad (4.6)$$

and  $\mathcal{L}_I$  is the usual WZ interaction Lagrangian. The situation here is similar to the case of  $\mathcal{N} = 1/2$  SUSY [31], where also the final effect of the deformation in the WZ model is the addition of a single interaction term involving the auxiliary field  $F$  in the action.

Despite our construction being formally covariant, to ensure the physical consistency we have to project the superfield action in term of the (physical) component fields, and verify explicitly the SUSY invariance. We start with the quadratic terms in eq. (4.1), and using eq. (3.21), we can write

$$\begin{aligned} \delta_\xi^* (\tilde{D}^b \triangleright \Phi \star \tilde{D}_b \triangleright \Phi) &= \delta_\xi^* (\tilde{D}^b \triangleright \Phi) \star \tilde{D}_b \triangleright \Phi + \tilde{D}^b \triangleright \Phi \star \delta_\xi^* (\tilde{D}_b \triangleright \Phi) + \\ &+ \frac{i}{2} C^{mn} \xi^a \left( \partial_m (\tilde{D}^b \triangleright \Phi) \star \partial_{na} (\tilde{D}_b \triangleright \Phi) + \partial_{ma} (\tilde{D}^b \triangleright \Phi) \star \partial_n (\tilde{D}_b \triangleright \Phi) \right). \end{aligned} \quad (4.7)$$

Because this expression is integrated we can use the property (4.2), together with eq. (3.19), to obtain

$$\begin{aligned} \delta_\xi^* (\tilde{D}^b \triangleright \Phi \star \tilde{D}_b \triangleright \Phi) &= \delta_\xi^* (D^b \Phi) \cdot D_b \Phi + D^b \Phi \cdot \delta_\xi^* (D_b \Phi) + \\ &+ \frac{i}{2} C^{mn} \xi^a \left( \partial_m D^b \Phi \cdot \partial_{na} D_b \Phi + \partial_{ma} D^b \Phi \cdot \partial_n D_b \Phi \right). \end{aligned} \quad (4.8)$$

It is easy to verify that the two terms in the second line of the previous equation cancel among each other, therefore,

$$\begin{aligned} \delta_\xi^* (\tilde{D}^b \triangleright \Phi \star \tilde{D}_b \triangleright \Phi) &= \delta_\xi^* (D^b \Phi) \cdot D_b \Phi + D^b \Phi \cdot \delta_\xi^* (D_b \Phi) \\ &= 2\delta_\xi^* (D^b \Phi) \cdot D_b \Phi, \end{aligned} \quad (4.9)$$

where eq. (3.20) was used in the last line. This final expression is clearly SUSY invariant. The same procedure can be used to argue for the SUSY invariance of the remaining quadratic term  $\Phi \star \Phi$ .

However, upon explicit calculation, the additional interaction term in eq. (4.5) is not invariant under the deformed SUSY transformation. To verify that, we remember that the integration over Grassmannian coordinates amounts to projecting the last component, proportional to  $\theta^2$ , of the integrand. So, SUSY invariance of eq. (4.5) means that the  $\theta^2$  component of  $\delta_\xi^* (\Phi \star \Phi \star \Phi)$  should be at the most a surface term. The action of  $\delta_\xi^*$  is distributed among the factors in the star product by the deformed Leibnitz rule, eq. (3.21). Using also eq. (4.2), we have

$$\begin{aligned} \delta_\xi^* (\Phi \star \Phi \star \Phi) &= (\delta_\xi^* \Phi) \star \Phi \star \Phi + \Phi \star (\delta_\xi^* \Phi) \star \Phi + \Phi \star \Phi \star (\delta_\xi^* \Phi) \\ &+ \frac{i}{2} C^{mn} \xi^a \Phi \star \left( \partial_m \Phi \star \partial_{na} \Phi - \partial_{ma} \Phi \star \partial_n \Phi \right) \\ &+ \frac{i}{2} C^{mn} \xi^a \left( \partial_m \Phi \star \partial_{na} (\Phi \star \Phi) - \partial_{ma} \Phi \star \partial_n (\Phi \star \Phi) \right). \end{aligned} \quad (4.10)$$

Also because of (4.2), all the terms in the first line of this last equation are equal. For the second and third lines, explicit expansion of the star products yields

$$\frac{i}{2} C^{mn} \xi^a \Phi \star \left( \partial_m \Phi \star \partial_{na} \Phi - \partial_{ma} \Phi \star \partial_n \Phi \right) = \frac{i}{2} C^{mn} \xi^a C^{pq} (\Phi \cdot \partial_p \partial_m \Phi \cdot \partial_q \partial_{na} \Phi), \quad (4.11)$$

while the third line vanishes. Therefore

$$\delta \left( \Phi \star \Phi \star \Phi \right) \Big|_{\theta^2} = 3 \left( \delta_\xi^* \Phi \right) \star \Phi \star \Phi \Big|_{\theta^2} + \frac{i}{2} C^{mn} \xi^a C^{pq} (\Phi \cdot \partial_p \partial_m \Phi \cdot \partial_q \partial_{na} \Phi) \Big|_{\theta^2}. \quad (4.12)$$

Finally, using eqs. (3.4), (3.20) and (2.12), projecting out only the terms proportional to  $\theta^2$ , after some algebraic manipulations, we arrive at

$$\begin{aligned} \delta \left( \Phi \star \Phi \star \Phi \right) \Big|_{\theta^2} &= \left[ 3i \xi^b \partial_a^b (\psi_b \cdot A^2) - \frac{3}{4} i C^2 \xi^a \partial_a^b \psi_b \cdot F^2 \right] + \\ &+ \frac{i}{4} C^{mn} C^{pq} \epsilon_{pm} \xi^a \left[ 2 \partial_{na} \psi_q \cdot F^2 - \psi_q \partial_{na} F^2 \right]. \end{aligned} \quad (4.13)$$

We observe that apart from the standard term, which is invariant, the NAC introduces additional ones, proportional to  $C^2$ , which do not appear as surface terms. For this reason the model is not invariant under deformed SUSY.

This lack of SUSY invariance is a surprise in this formalism since, differently from the  $\mathcal{N} = 1/2$  case in  $(3+1)$  dimensions [31], or also the three-dimensional deformations studied in [35, 38], for the twist we are considering the deformation is introduced in a way that the covariance of superfields and the algebra of supercharges is not deformed. However, this lack of SUSY invariance in the component formulation, despite the formally covariant construction was already pointed out in  $(3+1)$  dimensions [43, 45, 53, 54], where the twist formalism was also used. There, the lack of invariance was attributed to the need of introducing non-local projection operators to maintain the notion of chirality. Indeed, in the four-dimensional case, one has to apply (anti)chiral projectors,

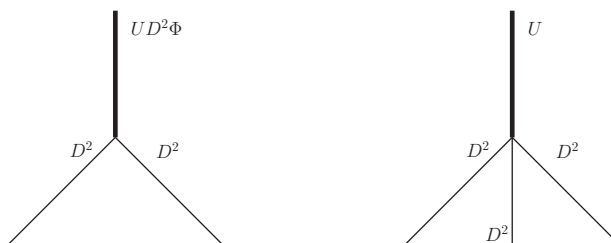
$$P_1 = \frac{1}{16} \frac{D^2 \bar{D}^2}{\square}, \quad P_2 = \frac{1}{16} \frac{\bar{D}^2 D^2}{\square}, \quad (4.14)$$

to star products of (anti)chirals superfields to maintain (anti)chirality. However, this introduces an ambiguity in the definition of the deformed trilinear interaction term, since for example both  $P_2 (\Phi \star \Phi \star \Phi)$  and  $P_2 (\Phi \star P_2 (\Phi \star \Phi))$  are acceptable, however upon explicit calculation, it was shown that the first expression is not SUSY invariant, while the second is. This is rather surprising since both are formally covariant.

We find that, even in the absence of non-local projection operators, the formal covariance of the superfield action is not enough to guarantee the SUSY invariance of the model. This seems to be a rather important shortcoming of the Hopf algebra formalism in the definition of consistent deformed superfield theories.

## 5 Quantum properties and renormalization

Despite the problem with SUSY invariance unveiled in the previous section, one may still wonder about the quantum properties of the model defined by eq. (4.1). Indeed, this model



**Figure 1.** New vertices arising from the  $U$ -term. Thin lines represent the quantum field, which appears only in internal lines of the diagrams.

might still define a consistent, yet non-supersymmetric, quantum field theory. Functional quantization of eq. (4.1) is possible by means of the methods similar to the ones used in the context of  $\mathcal{N} = 1/2$  models in [32, 33]: the NAC deformation amounts to the addition of the single (non-invariant) term in the action, and this could be incorporated in the superfield formalism by means of a spurion field given by

$$U(z) = -\det C \theta^2, \quad (5.1)$$

where we used the definition (4.6). In this way, we may rewrite eq. (4.5) as follows,

$$\mathcal{S}^* = \int d^3x d^2\theta \left[ \frac{1}{2} \Phi (D^2 + m) \Phi + \frac{\lambda}{6} \Phi^3 - \frac{\lambda}{24} U (D^2 \Phi)^3 \right]. \quad (5.2)$$

After we have written the action in this form, we can use the standard tools of superspace perturbation theory. We start by using the background method, splitting the superfield into its classical and quantum parts,

$$\Phi \rightarrow \Phi + \Phi_q, \quad (5.3)$$

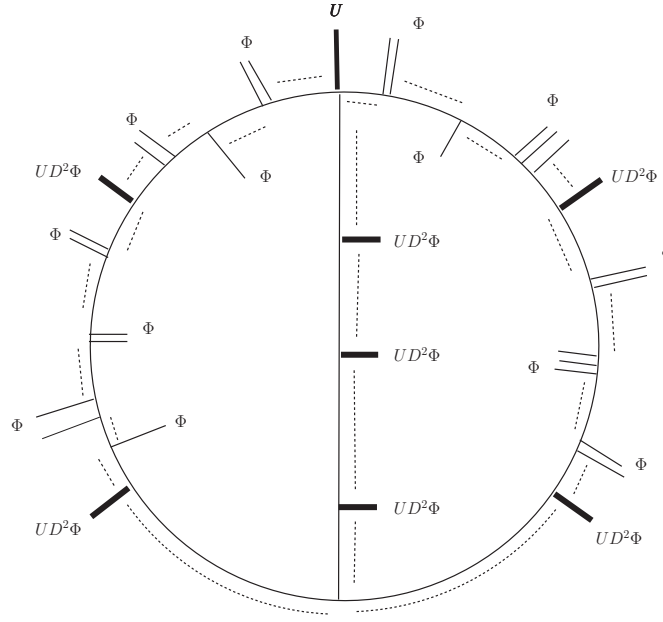
and then integrating over the quantum superfields  $\Phi_q$  in the path integral. The perturbative expansion involves the usual Feynman rules of the three-dimensional WZ model, the propagator given by

$$\langle \Phi \Phi \rangle = \frac{D^2 - m}{k^2 + m^2} \delta(\theta - \theta'), \quad (5.4)$$

the trilinear vertex factor corresponding to the coupling constant  $\lambda$ , together with two additional vertices involving the spurion  $U$ , represented in figure 1, which corresponds to a factor of  $\lambda/4$ .

Using regularization by dimensional reduction [55], one loop diagrams are finite, so we study the possibly divergent diagrams at the two-loop level. We follow the general strategy of [32], looking for the superficial degree of divergence of a general diagram containing several possible insertion of the quartic  $U$ -vertices. We have to consider three distinct classes of diagrams, represented in figures 2, 3 and 4, according to the number of quartic vertices including the spurion.

We start with the class of two-loop graphs represented in figure 2. These diagrams contains  $k$   $\Phi^3$ -vertices, and the total number of spurion vertices is  $p$  (one of them being the



**Figure 2.** Two loop diagram with one insertion of the quartic  $U$ -vertex.

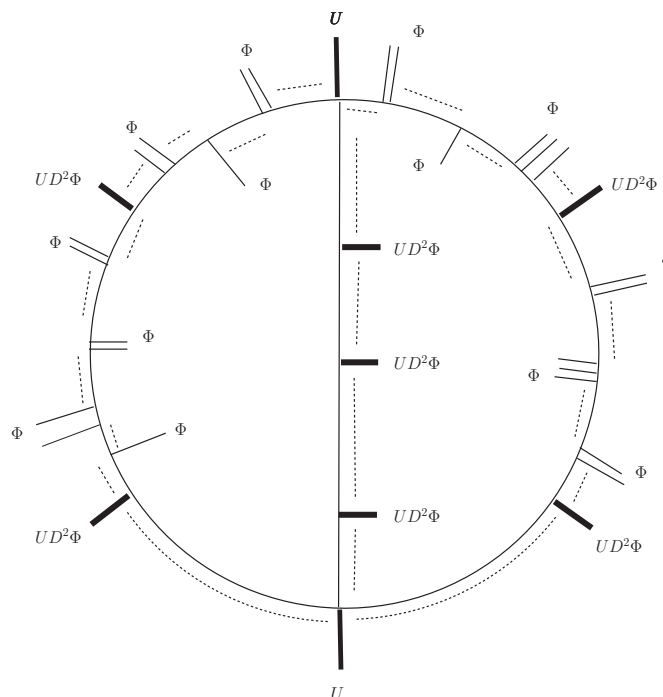
quartic  $U$  vertex, and the remaining  $p - 1$  being trilinear  $U D^2 \Phi$  vertices). To study the possible divergent configurations we compute the mass dimensions of the corresponding integrals once the  $D$ -algebra has been performed, for this we need to know the number of  $D$ 's and propagators in the graph, always considering the most divergent configuration. We take into account that we generate momentum factors through the algebraic relations

$$(D^2)^2 = \square, \quad D^2 D_m = P_{mb} D^b. \quad (5.5)$$

The number of  $D^2$  factors in the numerator is calculated considering the that each  $U$ -vertex contributes with three factors of  $D^2$ , while the  $U D^2 \Phi$ -vertex contributes with two factors of  $D^2$ , and each propagator with one factor of  $D^2$ . The number of propagators for this first class of diagrams is  $k + p + 2$ . Since for this class, we have one  $U$ -vertex and  $(p - 1)$   $U D^2 \Phi$ -vertex, the power of  $D^2$  in the numerator of the Feynman amplitude is given by  $3 + 2(p - 1) + k + p + 2 = 3p + k + 3$ . Since the spurion  $U$  has only the  $\theta^2$  component, we have to move a factor of  $D^2$  onto  $(p - 1)$   $U$  factors to obtain a final expression different from zero. We also use a factor of  $D^2$  to contract each loop to a point. Therefore, the number of remaining factors of  $D^2$  will be  $2p + k + 2$ . These  $D^2$  factors will lead to powers of momenta in the numerator according to eq. (5.5). Finally, taking into account the denominators of the propagators, we end up with a final integrand of the general form

$$\int \frac{d^6 q}{q^{k+2}}, \quad (5.6)$$

which by power counting is divergent if  $k \leq 4$ . This condition does not depend of  $p$ , therefore for each fixed  $k$  we have an arbitrary number of  $U$  insertions, all of them being



**Figure 3.** Two loop diagram with two insertions of the quartic  $U$ -vertex.

in principle superficially divergent. That means the model has an infinite number of potentially divergent diagrams, which suggests non-renormalizability. This result is different from that in four dimensions [32], where the value of  $p$  which could yield a divergent diagram was bounded, so the number of potentially divergent diagrams was finite. The notion of chirality, essential to four-dimensional SUSY, imposes additional conditions on the manipulation of covariant superderivatives, such that actually one can just have a divergent diagram with at the most a single  $U$  insertion.

For the diagrams represented in figure 3, containing two quartic  $U$  vertices, the number of propagators and initial  $D^2$  factors are, respectively,  $k + p + 1$  and  $3p + k + 3$ . Then one can show these diagrams will behave like

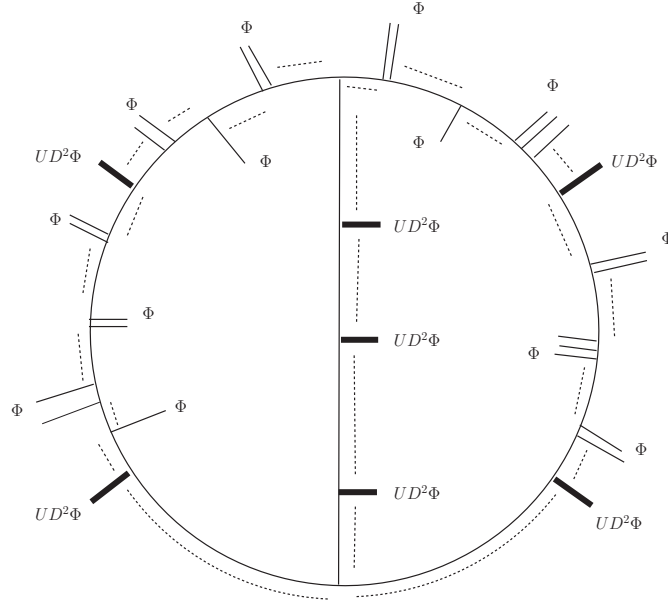
$$\int \frac{d^6 q}{q^k}, \quad (5.7)$$

which will be divergent if  $k \leq 6$ , again irregardless of  $p$ . Finally, for diagrams like the one depicted in figure 4, containing no quartic  $U$  vertices, we start with  $(3p + k + 3)$   $D^2$  factors and  $k + p + 3$  propagators, and the final integrand ends up being of the form

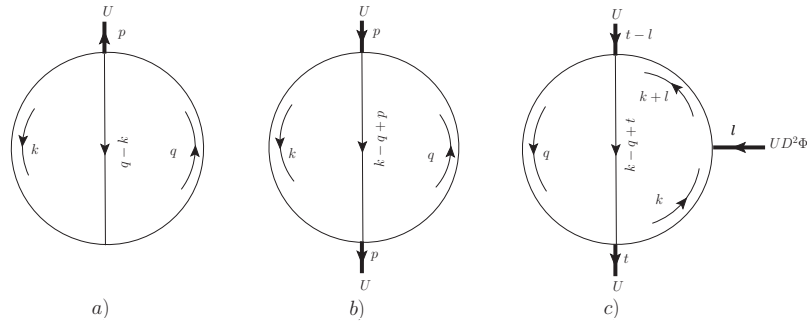
$$\int \frac{d^6 q}{q^{k+4}}, \quad (5.8)$$

which will be divergent if  $k \leq 2$ , again for arbitrary number of trilinear  $U$  vertices.

These results indicate that, unless some unexpected cancellation or additional constraints can be shown to occur in the infinite number of potentially divergent diagrams, we



**Figure 4.** Two loop diagram without insertions of the quartic  $U$ -vertex.



**Figure 5.** Examples of divergent supergraphs involving one, two and three spurion insertions.

have a non-renormalizable model. To show explicitly that these cancellations do not happen in general, we calculate the diagrams presented in figure 5, which involves one, two and three insertions of the spurion field, always with  $k = 0$ . According to the power counting derived in this section, these diagrams could have divergences of order four (diagram 5a) and six (diagrams 5b and c), and we will show that these divergences reduce to quadratic and logarithmic, but still are present in the final result.

From the Feynman rules described in this section, we write the amplitudes corresponding to the diagrams in figure 5, and perform all the integration by parts and simplifications involving the covariant derivatives with the use of the *Mathematica* package *SusyMath* [56].



The resulting amplitudes, using in the dimensional regularization scheme, are given by

$$\Gamma_a = \frac{\lambda^2}{16} \int d^2\theta \int \frac{d^3p}{(2\pi)^3} \int \frac{d^Dk d^Dq}{(2\pi)^{2D}} \left( \frac{-2m^2q \cdot k - m^3D^2 + 2m^4}{[k^2 + m^2][q^2 + m^2][(q-k)^2 + m^2]} \right) U, \quad (5.9)$$

$$\Gamma_b = \frac{3}{64} \int d^2\theta \int \frac{d^3p}{(2\pi)^3} \int \frac{d^Dk d^Dq}{(2\pi)^D} \left( \frac{m^6}{[k^2 + m^2][q^2 + m^2][(q-k)^2 + m^2]} \right) U D^2 U, \quad (5.10)$$

and

$$\Gamma_c = \frac{15}{32 \times 64} \lambda^3 \int d^2\theta \int \frac{d^3p}{(2\pi)^3} \int \frac{d^Dk d^Dq}{(2\pi)^D} \left( \frac{-3m^6}{[k^2 + m^2][q^2 + m^2][(q-k)^2 + m^2]} \right) \times \\ \times D^2 U D^2 U D^2 \Phi(p) U, \quad (5.11)$$

where  $D = 3 - \epsilon$ . The expression in eq. (5.9) contains quadratic and logarithmic divergences, while eqs. (5.10) and (5.11) contains logarithmic divergences. After evaluation of the momentum integrals, the explicit divergent contributions we found can be cast as

$$\Gamma_a^{\text{div}} = \frac{\lambda^2}{16} \left( \frac{m^3}{32\pi^2\epsilon} \right) \int d^2\theta \int \frac{d^3p}{(2\pi)^3} (D^2 - m) U, \quad (5.12)$$

$$\Gamma_b^{\text{div}} = \frac{3}{64} \lambda^2 \left( -\frac{m^6 C^2}{32\pi^2\epsilon} \right) \int d^2\theta \int \frac{d^3p}{(2\pi)^3} U, \quad (5.13)$$

and

$$\Gamma_c^{\text{div}} = \frac{15}{32 \times 64} \lambda^3 \left( \frac{3m^6}{32\pi^2\epsilon} C^4 \right) \int d^2\theta \int \frac{d^3p}{(2\pi)^3} U D^2 \Phi(p). \quad (5.14)$$

These examples show that, despite the fact that the power counting is overly pessimistic in general, there are still divergences present even for higher than one insertions of the spurion field. This is in contrast with the results found in  $\mathcal{N} = 1/2$  models [32, 33] in four dimensions, despite the fact that in our case the final effect of the deformation in the classical action is also the inclusion of a single additional interaction term. Despite the similarities in the diagrammatic expansion of both cases, the existence of the notion of chirality in four space-time dimensions imposes additional constraints which contributes to ensure renormalizability. Finally, it is also interesting to remark that WZ models in  $(3+1)$  dimensions deformed by means of a Drinfel'd twist were also shown to be non-renormalizable [43, 45, 53, 54], similarly to what we found here.

## 6 Concluding remarks

The deformation of supersymmetric models using the concept of a Drinfel'd twist preserves several important algebraic properties of the SUSY algebra, and it could allow for the definition of deformed supersymmetric models with interesting properties. However, when trying to put the Hopf algebraic formalism to work in a specific physical model, one often

encounters difficulties. In [43, 46], for example, a particular twist in four space-time dimensions was considered, and it was shown that even a formally supersymmetric covariant action involving superfields could fail to be SUSY invariant, when projected in terms of the component fields. In this case, the notion of chirality seems to be responsible for these problems, since it forces one to introduce non-local projection operators in the formalism. Also, a simple generalization of the WZ model failed to be renormalizable.

In three space-time dimensions, there is no notion of chirality, so in principle the application of the Drinfel'd twist would be simpler, and could open up the possibility of studying deformed  $\mathcal{N} = 1$  supersymmetric models, which is difficult to do in other formalisms. Therefore, we studied a twist deformation of three-dimensional  $\mathcal{N} = 1$  SUSY, and defined what would be the simplest non-anticommutative WZ model in this context. However, we showed that this theory suffers from the same problems present in the four-dimensional case. At the classical level, the model fails to be invariant under deformed SUSY transformations, meaning that although the star product and the deformed co-product are algebraically compatible, this compatibility does not guarantee actual SUSY invariance of the physical model under consideration. At the quantum level, the WZ model is finite at the one loop level, but at two loops there are an infinite number of potentially divergent diagrams, rendering the model non-renormalizable.

Our results reinforces the idea that algebraic consistency does not, by itself, guarantee the definition of physically meaningful deformed models, either at the classical or at quantum level. We studied a very simple twist deformation, and one might conjecture whether there are more complicated twists that could lead to consistent theories. This is a problem that we leave for future studies.

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